Moduli Spaces of ℝeal and ℂomplex Varieties

ANGERS (FRANCE), June 2–6 2014

Program 1/5: Mini-courses

Fabrizio Catanese, Bayreuth (Germany)
Topological methods in moduli theory

Projective varieties which are $K(\pi, 1)$'s. Group actions on Abelian varieties and Bagnera-De Franchis varieties.
Orbifold fundamental groups and moduli spaces of curves with symmetries.
Group cohomology and homology, topological invariants of group actions on curves.
Inoue type manifolds and surfaces and their moduli spaces.

Some reference:
1. F. Catanese – Differentiable and deformation type of algebraic surfaces, real and symplectic structures, in *Symplectic 4-manifolds and algebraic surfaces*, Lecture Notes in Math., vol. 1938, Springer, Berlin, 2008, p. 55–167.
2. F. Catanese – A Superficial Working Guide to Deformations and Moduli, in Handbook of moduli I, Gavril Farkas and Ian Morrison eds., Advanced Lectures in Mathematics, Volume 24, International press (2013), pages 161–216, `arXiv:1106.1368`.
3. I. Bauer, F. Catanese – Inoue type manifolds and Inoue surfaces: a connected component of the moduli space of surfaces with $K^2 = 7, p_g = 0$. Geometry and arithmetic, 23–56, EMS Ser. Congr. Rep., Eur. Math. Soc., Zürich, 2012.
4. F. Catanese, M. Lönne, F. Perroni, – Genus stabilization for moduli of curves with symmetries `arXiv:1301.4409`.
5. F. Catanese – Topological methods in moduli theory, in preparation.

Radu Laza, Stony Brook (USA)
Lectures on the moduli of cubics

In these lectures, I will discuss the construction and structure of moduli spaces for some special classes of varieties (most notably for cubic fourfolds). The common feature of the examples discussed here is that these moduli spaces have a dual description: GIT and Hodge theoretic (coming from a VHS of K3 type). I will discuss each of these approaches individually, and then describe how they fit together and the advantages of having both. More specifically, in the first lecture I will discuss the Looijenga 's framework of comparing the GIT and Hodge theoretical realizations of certain moduli spaces. Then, in lectures 2 and 3, I will discuss in some detail the moduli spaces of cubic fourfolds and cubic threefolds.

Some references:
1. Perspectives on the construction and compactification of moduli spaces, `arXiv:1403.2105`.
2. GIT and moduli with a twist, in "Handbook of Moduli" vol. 2, Adv. Lect. Math. 25 (2013), Int. Press, 259–297.
3. The moduli space of cubic fourfolds via the period map, Ann. of Math. 172 (2010), no. 1, 673–711.
4. The moduli space of cubic threefolds via degenerations of the intermediate Jacobian (w. S. Casalaina-Martin), J. Reine Angew. Math. 633 (2009), 29–65.
5. Log canonical models and variation of GIT for genus four canonical curves, (w. S. Casalaina-Martin and D. Jensen), to appear in J. Algebraic Geom. `arXiv:1203.5014`.

Moduli Spaces of $\mathbb{R}$eal and $\mathbb{C}$omplex Varieties

ANGERS (FRANCE), June 2–6 2014

Program 2/5: Talks

Miguel-Angel Barja, Barcelona (Spain)
The generalized Clifford-Severi inequality

Classical Severi inequality is a sharp lower bound for the self-intersection of the canonical divisor of a maximal albanese dimension surface. It was proved in full generality by Pardini some years ago who deduced it from the slope inequality. Since then, the lack of a higher dimensional slope inequality was an obstruction to generalize Severi inequality for higher dimensional irregular varieties. I will explain how to extend it to any dimension and to any nef line bundle on an irregular variety by a suitable use of an induction process. It provides an useful tool for the study and classification of irregular varieties.

Ingrid Bauer, Bayreuth (Germany)
The classification and moduli space of Generalized Burniat Type Surfaces

Primary Burniat Surfaces were constructed by P. Burniat in the sixties as singular bidouble covers of the projective plane. They are surfaces of general type with $K^2 = 6$ and geometric genus zero. In '94 M. Inoue gave another description of these surfaces as the étale $(\mathbb{Z}/2\mathbb{Z})^3$ quotients of an explicitly given invariant hypersurface X of multidegree $(2, 2, 2)$ in the product of three elliptic curves $E_1 \times E_2 \times E_3$.
We generalize this construction and classify completely the étale $(\mathbb{Z}/2\mathbb{Z})^3$ quotients S of an invariant hypersurface X of multidegree $(2, 2, 2)$ (which is the pullback of an irreducible Del Pezzo surface in $(\mathbb{P}^1)^3$ in the product of three elliptic curves $E_1 \times E_2 \times E_3$. We get 16 families of surfaces of general type with $0 \leq p_g = q \leq 3$. Moreover, we describe the connected components of the Gieseker moduli space to which these surfaces belong. In the case $p_g = q = 0$ we are able to verify Bloch's conjecture.
This is a joint work with F. Catanese and D. Frapporti.

Laurent Évain, Angers (France)
Nested punctual Hilbert schemes and equivariant Chow rings

The irreducibility of the punctual Hilbert scheme in the plane $Hilb_0^n(\mathbb{A}^2)$ has been first proved by Briançon. An alternative proof relies on the geometry of the variety parametrizing pairs of commuting nilpotent matrices. We extend this circle of ideas to describe the geometry of nested Hilbert schemes $Hilb_0^{n,m}(\mathbb{A}^2)$ parametrizing pairs (z_n, z_m) of punctual subschemes with $z_n \subset z_m$.
As an application, we construct creation operators on the equivariant Chow ring $A_K^*(Hilb^n(\mathbb{A}^2))$ of the Hilbert scheme $Hilb^n(\mathbb{A}^2)$ with equivariant coefficients inverted. We compute base change formulas in $A_K^*(Hilb^n(\mathbb{A}^2))$ between the natural bases introduced by Nakajima, Ellingsrud and Strømme, and the classical basis associated with the fixed points.

Moduli Spaces of $\mathbb{R}$eal and $\mathbb{C}$omplex Varieties

ANGERS (FRANCE), June 2–6 2014

Program: 3/5: Talks

Sergey Finashin, Ankara (Turkey)
Apparent contours of real cubic surfaces

In a joint work with V. Kharlamov we obtained a complete deformation classification of real Zariski sextics, that is of generic apparent contours of nonsingular real cubic surfaces. Such sextics have singular points: six cusps lying on a conic. The classification is obtained by studying the moduli of the K3-surfaces being the double planed branched along the Zariski sextics.
As a by-product, we observe a certain "reversion" duality in the set of deformation classes of these sextics. This kind of duality can be also observed for non-singular real sextics (in which case it simply interchanges the "internal" and "external" ovals), but in presence of singularities it turns into a more sophisticated correspondence.

Ilia Itenberg, Paris
Hurwitz numbers for real polynomials

We introduce a signed count of real polynomials which gives rise to a real analog of Hurwitz numbers in the case of polynomials. The invariants obtained allow one to show the abundance of real solutions in the corresponding enumerative problems: in many cases, the number of real solutions is asymptotically equivalent (in the logarithmic scale) to the number of complex solutions. This is a joint work with Dimitri Zvonkine.

Yoichi Miyaoka, Tokyo (Japan)
The Mehta-Ramanathan theorem and the Bogomolov inequality for semistable Higgs bundles

We give an algebraic proof of the following three basic results on Higgs bundles:

1. The restriction of a semistable Higgs bundle to a general, sufficiently ample hypersurface is semistable.
2. The tensor product of two semistable Higgs bundles is semistable.
3. The 1st and the second Chern classes of a semistable Higgs bundle satisfies the Bogomolov inequality.

Viacheslav V. Nikulin, Liverpool (United Kingdom)
Degenerations of Kahlerian K3 surfaces with finite symplectic automorphism groups

We use our recent results about Kahlerian K3 surfaces and Niemeier lattices to classification of degenerations of Kahlerian K3 surfaces with finite symplectic automorphism groups.
See my recent preprint `arXiv:1403.6061` for details.

Moduli Spaces of $\mathbb{R}$eal and $\mathbb{C}$omplex Varieties

ANGERS (FRANCE), June 2–6 2014

Program 4/5: Talks

Gianpietro Pirola, Pavia (Italy)
Isogenies between Jacobians

We will discuss a basic problem concerning the correspondences of curves. Fixed an integer $g > 1$, let $M(g)$ be the moduli space of the complex smooth complete curves of genus g. We define the isogeny locus $H \subset M(g) \times M(g)$ defined by the curves with isosogeneus Jacobians. Many years ago (in 1989), with Fabio Bardelli, we proved that if $g > 3$, then the diagonal D is the only component of H of dimension $3g - 3$. Recently, in a joint work with Valeria Marcucci and Juan Carlos Naranjo, we proved
Theorem Set $B = H - D$. Then

1. If $g > 3k + 4$, then $\dim B < 3g - 3 - (k - 1)$,
2. if $g > 4$ then $\dim B < 3g - 4$.

The proof of the first part follows an idea of Claire Voisin. One uses infinitesimal variations of Hodge structures in order to translate the statement into a problem concerning the quadrics containing a canonical curve. The second part of the proof consists to degenerate the isogenies to some special singular stable curves. This second approach needs a good control of the intersection with the boundary the Deligne-Mumford compactification of $M(g)$.

Xavier Roulleau, Poitiers
Geography of simply connected surfaces of general type"

The Chern numbers c_1^2, c_2 of a smooth minimal surface of general type X satisfy the Bogomolov-Miyaoka-Yau inequality: $c_1^2 \leq 3c_2$. Thirty-five years ago, Bogomolov asked if one can improve the BMY inequality to $c_1^2 \leq ac_2$ with $a < 3$ when we moreover suppose that X is simply connected. In this talk, we will show that there exists spin (resp. non-spin) simply connected surfaces with c_1^2/c_2 arbitrarily close to 3, and therefore the answer is negative. This is a joint work with Giancarlo Urzúa.

Moduli Spaces of $\mathbb{R}$eal and $\mathbb{C}$omplex Varieties

ANGERS (FRANCE), June 2–6 2014

Program 5/5: Talks

Roberto Pignatelli, Trento (Italy)
Rigid Ample Divisors on some Calabi-Yau 3-folds

The first example of a Calabi-Yau 3-folds with nonabelian fundamental group has been constructed by A. Beauville in 1994 inspired by M. Reid example of a numerical Campedelli surface, surface which is indeed contained in the 3-fold as rigid ample divisor. In the same note Beauville notices that a smooth rigid ample divisor in a Calabi-Yau 3-fold is a surface of general type of genus zero. In this talk I will discuss some examples of Calabi-Yau 3-folds containing a rigid ample divisor and their relevance from the point of view of the theory of the surfaces of general type.

Rares Rasdeaconu, Nashville (USA)
Counting real rational curves on K3 surfaces

Real enumerative invariants of real algebraic manifolds are derived from counting curves with suitable signs. I will discuss the case of counting real rational curves on K3 surfaces equipped with an anti-holomorphic involution.
An adaptation to the real setting of a formula due to Yau and Zaslow will be presented. The proof passes through results of independent interest: a new insight into the signed counting, and a formula computing the Euler characteristic of the real Hilbert scheme of points on a K3 surface, the real version of a result due to Gottsche.
The talk is based on a joint work with V. Kharlamov.

Igor Reider, Angers
Nonabelian Jacobian of smooth projective surfaces

The nonabelian Jacobian $\mathbf{J}(X; L, d)$ of a smooth projective surface X is inspired by the classical theory of Jacobian of curves. As its classical counterpart it is related on the one hand to the Hilbert schemes of points on X and on the other hand to the vector bundles (of rank 2 this time - here resides the nonablian aspect of the theory) on X. But it also relates to such influential ideas as variations of Hodge structures, period maps, nonabelian Hodge theory, Homological mirror symmetry, perverse sheaves, geometric Langlands program. These relations manifest themselves by the appearance of the following structures on $\mathbf{J}(X; L, d)$:
1) a sheaf of reductive Lie algebras,
2) (singular) Fano toric varieties whose hyperplane sections are (singular) Calabi-Yau varieties,
3) trivalent graphs.
In my talk I will explain the appearance of those structures and give some illustrative examples.

Jean-Yves Welschinger, Lyon
Expected topology of random real algebraic submanifolds

Given a smooth projective manifold defined over the reals, what is the expected topology of a codimension k submanifold chosen at random? (as the vanishing locus of a random section of some rank k holomorphic vector bundle). I will explain how the L^2 estimates of Hörmander make it possible to tackle this question asymptotically and in particular to estimate its Betti numbers. This is a joint work with Damien Gayet.