Martine Babillot
Potential Theory for random walks on Lie groups

1.– Harmonic functions and the mean-value property

2.– Poisson Boundaries and Furstenberg Rigidity

3.– Renewal theory and Green functions

4.– Martin compactifications, some new results and some open questions

5.– Extension for algebraic groups defined over local fields

Les references essentielles sont:

O. L'article "source": H. Furstenberg: Poisson formula for semi-simple Lie groups, ANN. Math, 77 (1963) 377-428

1. Guivarc'h, Ji et Taylor: Compactifications of symmetric spaces, (Birkhauser)

2. M. Babillot, Asymptotics of Green Functions on a calss of solvable Lie Groups, Potential Analysis, vol 8, (1998), p. 69–100

3. L'article de survey de Vadim Kaimanovich: Boundaries of invariant Markov operators : the identification problem, London Math Soc. Lecture Notes Series, 228, (1996) 127–176

4. A. Raugi : Fonctions harmoniques positives sur certains groupes de Lie resolubles connexes. Bull. Soc. Math. France, vol 124 (1996) p. 649–684.

Philippe Bougerol
Mouvement brownien sur les espaces symetriques.

1- Marche aléatoire simple sur les arbres,

a. Loi des grands nombres, theoreme central limite

b. Etude asymptotique du pont

2- Mouvement brownien sur les espaces symetriques.

a. Description du Laplacien en coordonnees polaires et horocycliques.

b. Loi des grands nombres et theoreme central limite.

c. La boucle brownienne infinie sur une variete Riemannienne.

d. Descrition de la boucle infinie sur un espace symetrique

e. Description du pont Brownien.

References:

Babillot M., *A probabilistic approach to heat diffusion on symmetric spaces*, J. Theoret. Probab. **7** (1994), 599–607.

Bougerol Ph., *Comportement asymptotique des puissances de convolution d'une probabilité sur un espace symétrique*, Astérisque **74**, Soc. Math.

France (1980), 29–45.

Bougerol Ph. & Jeulin T., *Brownian bridge on hyperbolic spaces and on homogeneous trees*, Probab. Theory Relat. Fields, **115** (1999), 95–120.

Grabiner D.J., *Brownian motion in a Weyl chamber, non–colliding particles, and random matrices*, Ann. Inst. H. Poincaré **35** (1999), 177–204.

Guivarc'h Y., Ji L. & Taylor J.C., *Compactifications of symmetric spaces*, Progress Math. **156**, Birkhäuser (1998).

Helgason S., *Groups and geometric analysis*, Academic Press (1984).

Nechaev S.K., Grosberg A.Y. & Vershik A.M., *Random walks on braid groups : Brownian bridges, complexity and statistics*, J. Phys. Section A (*Math.*) **29** (1996), 2411–2434.

Nechaev S.K. & Sinaï Y.G., *Limiting–type theorem for conditional distributions of products of independent unimodular* 2×2 *matrices*, Bol. Soc. Brasil. Mat. **21** (1991), 121–132.

Sullivan D., *Related aspects of positivity in Riemannian geometry*, J. Diff. Geom. **25** (1987), 327–351.

Taylor J.C., *Brownian motion on a symmetric space of non–compact type : Asymptotic behavior in polar coordinates*, Can. J. Math. **43** (1991), 1–21.

Virtser A.D., *Central limit theorem for semi–simple Lie groups*, Theory Probab. Appl. **15** (1970), 667–687.

S.G. Dani

Asymptotic behavior of measures

1. Probability measures on vector spaces and projective spaces. Basic properties of limits of sequences of the form $A_i(\lambda_i)$, where $\{A_i\}$ is a sequence of linear maps and $\{\lambda_i\}$ is a sequence of probability measures.
2. Orbit spaces under actions of automorphism groups of linear groups. Extension to general connected Lie groups.
3. Convergence of types theorems; collapsible measures.
4. Study of factor sets of probability measures on Lie groups.
5. Decay of concentration functions of convolution powers.
6. Miscellaneous applications: Tortrat groups, Levy's measures etc.

References:

BOOKS:

K.R. Parthasarathy, Probability Measures on Metric Spaces, Academic Press, 1967

Chapter III, of R.J. Zimmer, Ergodic Theory and Semisimple Groups Birkhauser, 1984

PAPERS:

S.G. Dani and M. McCrudden, Factors, roots and embeddability of measures on Lie groups, Math. Zeits. 199 (1988), 369-385.

S.G. Dani, Invariance groups and convergence of types of measures on Lie groups, Math. Proc. Cambr. Phil. Soc. 112 (1992), 91-108.

S.G. Dani and Riddhi Shah, Collapsible probability measures and concentration functions on Lie groups, Math. Proc. Cambridge Phil. Soc. 122 (1997), 105-113.

S.G. Dani, Scattering of products of random matrices, In: Analysis, Geometry and Probability (ed: R. Bhatia), pp. 33-53, Hindustan Book Agency, New Delhi, 1996.

S.G. Dani, Actions on spaces of measures on Lie groups and convergence of types, In: Dynamical Systems and Related Topics: Collection of articles dedicated to Academician D.V. Anosov (Ed: V. Ryzhikov and A. M. Stepin), Proc. Steklov Inst. of Math. 256 (1997), 250-258.

S.G. Dani, Asymptotics of measures under group automorphisms (jointly with C.R.E. Raja), Proceed. of the Internat. Colloq. on Lie groups and Ergodic Theory, Tata Institute of Fundamental Research, Mumbai, pp. 59-73, Narosa Publishing House, New Delhi, 1998.

S.G. Dani and C.R.E. Raja, A note on Tortrat groups, J. Theor. Probability 11 (1998), 571-576.

MISCELLANEOUS POINTS:

The course is intended to develop the theme into an area in which participants can pursue research. Various possible future directions will be discussed. I propose also to write a book based on the notes of the lectures to be given at the workshop.

Jacques Faraut
Probabilités sur les cônes symétriques

Le cours sera consacré à l'exposition des résultats récents sur les lois de probabilité sur le cône des matrices symétriques et d'autres cônes symétriques: loi de Wishart, familles exponentielles des probabilités, loi de Dirichlet, la loi gaussienne inverse etc. Toutes ces lois ont des applications importantes en statistique multivariée.

REFERENCES

Casalis, M.; Letac, G. The Lukacs-Olkin-Rubin characterization of Wishart distributions on symmetric cones. Ann. Statist. 24 (1996), no. 2, 763–786.

Faraut, Jacques; Kornyi, Adam, Analysis on symmetric cones. Oxford Mathematical Monographs. Oxford Science Publications. The Clarendon Press, Oxford University Press, New York, 1994.

Letac, Gérard; Massam, Hélène, Quadratic and inverse regressions for Wishart distributions. Ann. Statist. 26 (1998), no. 2, 573–595.

Letac, Gérard; Massam, Hélène, A formula on multivariate Dirichlet distributions. Statist. Probab. Lett. 38 (1998), no. 3, 247–253.

Letac, Gérard; Massam, Hélène, All the moments and inverse moments of the Wishart distribution, preprint 1999.

Muirhead, Robb J., Aspects of multivariate statistical theory. Wiley Series in Probability and Mathematical Statistics. John Wiley & Sons, Inc., New York, 1982.

Yves Guivarc'h
Théorèmes limites
pour les produits de matrices aléatoires

I. Introduction

1) Le modèle d'Anderson.

2) La marche aléatoire sur $\mathbf{Z}$ en milieu désordonné.

3) La relation de récurrence $x_{n+1} = A_{n+1}x_n + b_{n+1}$.

II. Exposants des produits de matrices

1) Le plus grand exposant.

2) La simplicité du spectre de Liapunov.

3) Les propriétés de contraction sur l'espace projectif.

III. Le théorème central limite

1) La loi limite des vecteurs colonnes.
2) L'allure asymptotique de la décomposition polaire.
3) La non-dégénerence des lois limites et les équations cohomologiques.

IV. Les opérateurs transformés de Laplace

1) La normalisation des opérateurs P^s.
2) Les exposants du système Markovien associé.
3) Les propriétés spectrales des opérateurs P^s.

V. Relations de récurrence à coefficients aléatoires

1) Le théorème de renouvellement.
2) Les opérateurs P^s et l'allure à l'infini des lois stationnaires.

Bibliographie

1. Bougerol, Lacroix: Products of random matrices with applications to Schroedinger operators, Birkhauser, 1986.

2. Guivarc'h-Raugi: Products of random matrices. Convergence theorems. Random matrices, Cont. Math., AMS,50, p.31-54, 1986.

3. Furstenberg: Boundary theory and stochastic processes on homogeneous spaces, Proc. Symp. Pure

4. Kesten: Random difference equations and renewal theory for products of random matrices, Acta Math. 131, p.208-248, 1973.

5. Goldcheid-Guivarc'h: Zariski closure and the dimension of the Gaussian law of the product of random matrices, Prob.Th.Rel.Fields, 105, p.109-142, 1996.

6. Guivarc'h: Random walks on Lie groups, Tata Institute Lectures on Mathematics, 1999, to appear.

M. McCrudden

The embedding problem for probabilities on groups

Topics.

G denotes a locally compact group.

1.Factor compact measures on G.Factor compact groups.The Factor Compactness Theorem.

2.Convolution bounded groups.Every connected group G is convolution bounded.

3.Root compact measures, rational and continuous embedding,Boge root compactness.

4.The Embedding Theorem for

(a)linear p-adic groups,

(b)coverings of real connected Linear groups,

(c)discrete real linear groups.

5.Recent work on the extension of the theory from groups to semigroups.

References.

1.HEYER,H."Probability measures on locally compact groups" Springer-Verlag,
Berlin,1977.

2.Heyer,H."Recent contributions to the embedding problem for probability measures on a locally compact group," J.Multivariate Anal.19 (1986),119-131.

3.SHAH,R."Infinitely divisible measures on p-adic groups,J.Theor. Prob.4 (1991),391-405.

4.DANI,S.G. and McCRUDDEN,M."Embeddability of infinitely divisible measures on linear Lie groups. Invent.Math.110 (1992),237-261.

5.DANI,S.G. and McCRUDDEN,M."Infinitely divisible probabilities on discrete linear groups" J.Theor.Prob.9 (1996),215-229.

6.McCRUDDEN,M. and WALKER,S."Embedding infinitely divisible probabilities on subsemigroups of Lie groups" to appear in Comtemporary Math.(Proceedings).

Statistics in abstract spaces, negative probabilities, classical decompositions

Gábor J. Székely

The lecture series consists of three non-disjoint parts.

(i) Newton's potential energy is generalized for statistical applications. First, we introduce the notion of internal energy =capacity and external energy = potential energy =PE of statistical samples in abstract spaces. Then we construct nonparametric PE tests and estimators. They turn out to be very simple and powerful. It is shown that the sample PE is always nonnegative and converges to 0 iff the sample distribution converges to the null distribution. The notion of sample kinetic energy will also be discussed. The energy perspective of statistics, the principle of least possible effort = minimizing the energy is very appealing and also powerful in terms of effectiveness of statistical decisions.

(ii) Probabilities are numbers in the interval [0,1]. Although some of the results of probability theory can easily be generalized to signed measures, we do not really pay much attention to negative probabilities because we do not really see their significance, their applicability. Half of a fair coin is a good example. The sum of two independent half coins is a (complete) coin, thus the probability generating function of a half coin is $[(1+z)/2]^{1/2}$. So, a half coin takes the integer value $n = 0, 1, \ldots$ with probability $p_n = (-1)^{n-1}\sqrt{2}C_{n-1}/4^n$ where C_n is the n-th Catalan number. In this sequence every other probability is negative. Does this make any sense? In the following sense it does: signed, normed probability measures are always convolution quotients of (nonnegative) probability measures on 'arbitrary' topological groups.

(iii) Probability on algebraic structures is closely related to the algebra of probability distributions where the most frequently used operations between probability distributions (on algebraic structures) are considered from a purely algebraic (or rather topological algebraic) point of view, e.g. the convolution turns the set of probability measures (on a group) a semigroup (endowed with the topology of weak convergence). David Kendall's Delphic theory (1968) was the first attempt to construct a general algebraic framework to study convolutions. Twenty years later jointly with Imre Z. Ruzsa we built up a more general theory of Hun and Hungarian semigroups (one of Kendall's axioms turned out to be superfluous). Details can be found in our monograph 'Algebraic Probability Theory'. Since the publication of this book in 1988 many authors applied our general theory in concrete probabilistic structures. Further improvements were also published. The talk will summarize the main concepts and some of the new results of algebraic probability theory. New difficulties arrise when more than one operations are considered simultaneously.